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**Castell-fact algorithm: RSA, PGP, Bitcoin & Co.**

**25 partial considerations, using only multiplication and division to break down a composite product into its prime factors.**

**Consideration 20.**

**The right selection from the so-called starting aid list.**

**Concrete example for the application of the starting aid list, which will be presented in the eighth part of this essay:**

The two 2-digit numbers 23 \* 37 (or vice versa) belong to a number such as 851.

Whichever of these 2-digit products the algorithm assigns the function of the counting factor, this factor (to be calculated in the way used here) is always on the right-hand side and is made by means of its right-most digit, i.e. the 1st digit (here the “7”) the 2-digit factor on the left-hand side to the counted factor, which becomes a series of 7 through the determining “7” on the right-hand side. The latter cannot be recognized by its individual digits, but (as already explained several times) it is obvious when the decimal places belonging to the 7-series are added.

Then, for example, a “4” becomes “14”, which can be divided by “7” and the new factor is “2”, etc.

The knowledge of the decimal places belonging to a series of 7 is therefore necessary for factoring, but also to calculate the transfers that do not appear directly below in the total line (i.e. in the respective digit of the large number), but above it Sum row can be passed on to the column on the left.

So if the number 851 is kept ready for retrieval in the database of the algorithm, whereby it also supplies its two above-mentioned factors 23 and 37 (the very rare cases that two pairs of factors are supplied can be ignored. In the worst case, the same thing happens as it would have happened otherwise, without using this new system: “Stop and back and new beginning”), then only the first two factors are binding. In other words: Even if a 3rd or (with large numerical values) 4th digit appears in the result, these only largely have to do with the 3rd and 4th total digits with which they are compared. With two 2-digit factors (such as 23 and 37), only the first two digits of the resulting product (51) are binding. These two factors (calculated from the right) correspond exactly to the first two factors of the large number. A third digit, like the “8” here, changes with a further calculation. In a further calculation (if the multi-digit factor to be searched for is greater than 2-digit) it only represents the complete transfer from the 2nd column:

With the manual multiplication of 23 and 37, the first “1” would result in the 1st column and the two summands “6” and “9” in the 2nd column, which lead to the total number 5. Since the top line (with the “4” forms a 7-series, this “4” represents a “14” as an integer and results in a factorization factor of “2”, and the “9” in the second line underneath, which forms a row of 3, as the “3” on the left in “37” represents the new, ie second, counting factor for the 2nd line, is formed from 3 \* 3, which is a new, factoring factor of “3” means for the counting factor on the right.

As transfers result from the “15” of the first line in the 2nd column a “1”, then a “6” from the “2 \* 3” calculated in the 3rd column and a “1” from a sum transfer , because the 2nd total number was not 3, but “13” (total of 4 + 9). Everything together represents the “8” in the “851”, but this will be subtracted out of the 3rd column in the same way as the middle section that appears for the first time in the 3rd column. If there had been another (4th) digit to the left of the “8”, this would not have to be divided out from the 3rd but from the 4th column.

This middle section is easy to recognize and calculate. It is the product of the “neck” of the 1st line (this is the “2” of “23” here) and the counter from the 2nd line (this was the “3” of “37”). This product is “6” and does not generate a carryover to the 4th column, which would otherwise have to be divided out of the 4th column. The “6” is now calculated from the 3rd column, just like the above-mentioned transfer “8” (from 851).

Then the remaining 2 column digits of the 3rd column are found in the same way as was demonstrated in the appendixes part 8 and part 9 among others!

The transition from this “quick procedure” for the first two digits to the “normal” procedure with the double rows from the 3rd sum figure inclusive is therefore problem-free, since, as described above, any transfer (ie both that between the columns and that of one total digit to the next) from column 2 to column 3 is contained in the 3rd (and possibly 4th) digit of the 3 to 4-digit product from the first two 2-digit factors of the first two columns. The “8” described in example 851 or 817 is the transfer that is deducted in the 3rd column. The same applies to the middle part of the 3rd column, which is a product of 2 \* 3 = 6 in the 851 example or 1 \* 4 = 4 in the 817 example.

For the processing of the 3rd column, the transfers of the previous two total digits or columns cause no problems at all.

An inadequacy of this approach, however, is that the 3rd and possibly 4th digits of the two products obtained in the “rapid process” (851 and 817), here both times the “8”, are not identical to the 3rd. Sum digit, which is “3” in the case of the 817 (7643 \* 6719 = 51353317). Even if the above transfers (here “4” plus “8”) are subtracted from this total number “3” and only the total number “1” (or 11) remains, this “11” has no resemblance to the 3rd digit of the total line (ie the present large number), which is a “3” here.

In addition, within the total of 1,000 stored results, the calculation examples responsible for the exercise with the result 817 (from 19 \* 43) only concern 200 calculations (cf. 1 \* 7 and 3 \* 9) and those for the exercise with the result 851 (from 23 \* 37) relevant calculation examples 300 invoices (cf. 1 \* 1, 3 \* 7 and 9 \* 9), but that within these 300 or 200 invoices 10 percent are allotted to the desired endings. The latter means that in the 851 example there are 30 products in the amount of “51” and in the 817 example there are 20 products in the amount of “17”. Almost all of these 2-digit small products have different 3rd digits, but these say nothing about whether they correspond to the real 3rd digit of the large number. Even if the compared 3rd digits were identical, this fact would mean nothing, since the 3rd digits in the two sums were created for different reasons.

The aim here is therefore to find a method that enables a reliable selection from the 20 or 30 alternatives.

**Concrete example:**

For the above example 7643 \* 6719 = 51.353.317 the “quick method” is used and delivers the first two factor digits 43 \* 19 with the partial product 817, which arises when only the two 2 factor digits 43 and 19 are combined be multiplied.

The first two digits of the partial product 817 are binding and agree with the first two digits (from the right) of the large number (17).

However, the 3rd digit of the large number is “3” and no longer matches the “8” of the partial product. In fact, the “3” in the large number is an equivalent digit of this product, while the “8” of the partial product is only the entire transfer that accrued in the 2nd column and is now heading towards the 3rd column. The result is the decimal digit of the number at the top of the 2nd column (the “3” from 38 out of 4 \* 9 + the transfer “2” from 27 in the 1st column) plus the product of the bottom number (the “4”) from the multiplication of the two second factor numbers 4 \* 1) plus a “1” as a carryover in the sum, because the sum 8 + 3 in the 2nd column reaches the 2-digit tens range.

a)

The whole calculation:

7 6 4 3 \* 6 7 1 9 =

6 8 7 8 7

7 6 4 3

5 3 5 0 1

* 5 8 5 8

- - - - - - - - - -

5 1 3 5 3 **3** 1 7

b)

The part-calculation:

* \* 19 =

3 8 7

4 3

- - - -

**8** 1 7

Since this 3-part total transfer from the 2nd column in the 3rd column of the partial product (817) shows as “8”, it is not immediately apparent whether this 817 result is identical to the “317” in the large number is.

In fact, it is, because the 3rd column of the large number contains all 3 parts of the transfer “8”, which can be seen in the 3rd position in the partial product “817”:

The upper transfer “3” finds its way into the 54 from 4 \* 9 and makes it 57. The “4” below (from right to left) 4 \* 1 is also present and additionally the total transfer “1”. All together (3 + 4 + 1) results in the total transfer “8”, which is shown in the partial product 817.

If exactly these 3 transfers from the 2-digit factors 43 \* 19 were not present in the 3rd column, the actually present large number as the 3rd total number could not have the “3”. Nevertheless, it is not clear at the beginning to recognize the “correct” sub-product with the “correct” factor numbers from almost 20 products held in the database, all of which have the first two digits “17”.

The following are specifically available for selection:

17 (0**1** \* **1**7), 1917 (7**1** \* **2**7), 1517 (4**1** \***3**7), 517 (1**1** \* **4**7), 4617 (8**1** \* **5**7), 3417 (5**1** \* **6**7), 1617 (2**1** \* **7**7), 7917 (9**1** \* **8**7), 5917 (6**1** \* **9**7)

117 (1**3** \* **0**9), 817 (4**3** \* **1**9), 2117 (7**3** \* **2**9), 117 (**3** \* **3**9), 1617 (3**3** \* **4**9), 3717 (6**3** \* **5**9), 6417 (9**3** \* **6**9), 1817 (2**3** \* **7**9), 4717 (5**3** \* **8**9), 8217 (8**3** \* **9**9).

**Annotation:**

While the numbers on the top left are small partial products, the two 2-digit numbers on the right represent their factors. In order to reconstruct their original column numbers from these factors, the outer numbers of the two factors are always combined multiplies and results in the upper number of columns and then the inner and results in the lower number of columns.

Specifically: 43 \* 19

4 \* 9 = 36 (or 38 with transfer “2” from 3 \* 9 = 27) and 3 \* 1 = 3.

**Troubleshooting**

The problem is that there are almost 20 (with the “1” as the constant first digit also 30) solutions available, but it is not clear which of these two-digit 20 factors currently in development are the right ones. All 20 alternatives have “17” as the 1st and 2nd digits of the product “51.353.317”, but they differ from one another with their 3rd and possibly 4th factor numbers.

These 3rd and 4th factors represent the transfers contained in the 3rd and possibly 4th factor numbers of the actually large number. In the example given, “817”, “8” is the transfer, which cannot be seen in the 3rd digit of the large number. There is only a “3” here. This “3” is higher than the “8”, because the “8” is contained in the “3”. So there is a difference of “5” (13 - 8).

If this “5” is not just the result of “random” digits, but shows a regularity, this situation could be a possibility for the selection of the only correct alternative between the 20 offers in the list. So the question is, where is the “5”?

Answer: The “5” is made up of the sum transfer “1” (which arises from the sum “8” plus “3” in the 2nd column) and a “4” hidden in the 3rd column. In fact, this “4” is in the number “57” in the 1st line of the 3rd column, which arose from 6 \* 9, plus the transfer “3” from the 1st line of the 2nd column. In front of the transfer “3” there was a “54” in the 1st line of the 3rd digit, the one digit of which is the digit “4” you are looking for.

The following is a rule of thumb: The constant 1st digit “9” multiplied by the third digit of the counted factor (which was previously called “breast”) results in “4” (the 54 mentioned) in the 3rd column.

In the “quick process”, which only calculates the first two columns, this “4” in the 3rd column could not be produced (therefore, in the “817” formed in the quick process, only the “8” came about, but not the correct “ 3 ”or 13 for the really large number).

Since the digits of the 3rd column are not yet known, the question should be: “What can be multiplied by“ 9 ”and form the units digit“ 4 ”?” The solution is “54”, whereby the as yet unknown number in the 1st line of the 3rd column could only be “6”.

Using this above-mentioned procedure confirms the regularity that this procedure can also be used with the other 30 examples, but despite the incorrect numerical values ​​there (measured against the real large number), a consistent calculation results. In this respect, this procedure does not help to distinguish between the one correct and the 19 incorrect examples (with regard to the real large number).

The incorrect example 11 \* 47 = 517 with its “5” to the real correct “3” represents a difference of (13 - 5 =) “8”, or (since there is a sum transfer) “7” The above procedure results in this “7” as the unit's digit of the partial product when multiplying the first factor number “7”, which is constant for the first line, by the unknown 3rd digit x in the “counted factor” sought. This upper number in the 3rd column, which has a “7” as a one digit and is divisible by 7, is “7”, which means x = 1.

Since the 2nd column (for “17”) has a 1, there is a total transfer of “1”, although there is no transfer due to the constant factor numbers 1 \* 7 between the 1st and 2nd column.

The “8” missing in this example is therefore still a “7” after subtracting the total transfer “1” and is in the first row in the third position, i.e. in the third column, which is still in this so-called quick procedure is not recorded.

Whatever the simplest selection of the one correct calculation between the almost 20 available, it is definitely possible, so that the alternative procedure of this quick method is nevertheless recommended. According to the test divisions method proposed in Part 3 of the article on November 9, 2019, the calculation with additional factor numbers comes closer and closer to the ideal result. On the way there, after 1 to a maximum of 2 further columns, it becomes apparent which calculation comes closest to the result of the real large number in order to be the only one to reach it at the end.

The disadvantage of this method is that these experiments have to be carried out 20 times in parallel. The simplification of the effort that was initially aimed for has therefore not been achieved. However, the method has its justification in the speed of finding the first two factor digits and the certainty that the factor digits chosen at the beginning are definitely correct.

Tests:

**51 353 317 = 7643 \* 6719**

: 3 = 17.117.772, 3000

: 43 = 1.194.263, 1900

: 643 = 79.865, 1897

**: 7643 = 6.719, 0000**

Another advantage of the above procedure of using a start help list is that the initially conceived “guessing and possibly stopping and returning to the beginning” are no longer necessary.

With the list of numbers stored in the seventh part of this essay, all twenty (or in the case of the final prime number “1” thirty) possibilities are available at any time. These are 20 products that have the two final digits “17” to be examined (in this example chosen here).

Even “looking into” the underlying multiplications of these 20 products does not reveal an obvious pattern that proves which of these 20 products is the right one. On the other hand, there is only one result that is correct for each specific task at hand, even if 19 other calculations show the same result (here “17”).

In ignorance of the only correct calculation, all 20 cases at hand must be treated as if each of them were the correct one, i.e. for each of these cases the 3rd factor pair is produced separately. This results in 20 times 10 (= 200) results that all have the same 3 first digits “317”, i.e. all 200 existing calculations have the same first three total digits of the large number.

An owner of great computing power, who tends to transfer his own thinking to already known mathematics programs and possibly large computer systems, could now be tempted to let large work-stations work for himself instead of having his biological computer in his head obvious and simple solutions to come. If you have not tried this successfully in the last 4 decades, it may also be because of the prejudice propagated by the inventors (Ron Rivest etc.) that factoring really large numbers using a fast and simple algorithm is probably not possible.

To the spread of this not-feasible-myth also belongs the logically untenable often used comparison of his colleagues, in connection with factoring of the in-analysability of Coca-Cola or the irreversibility of a coffee-milk-mixture etc. From a logical point of view, a whole number does not break down into small, no longer identifiable, individual parts or changes “chemically”. Like cubes of equal size, described with a “one”, they are constantly re-forming and for the sake of simplicity they can also be called “three” instead of “1, 1, 1”, but they are always retained as whole numbers and can be hidden , move back and forth, rearrange, but also bring it back, etc.

Now, not everyone may follow the above picture, but it is helpful to first use linguistically appropriate terms and images before solving any problem, before one gets lost in logically wrong ideas of “cold coffee” with milk etc. With the linguistically correct images it is not far to the “linguistically pausable” certainty that numbers (equal-sized single cubes) that seem to have disappeared by multiplication can also be conjured up again with the help of suitable actions.

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The situation described above, that the first two digits of the large number for a certain pair of numbers (here “17”) provides 20 possibilities and for reaching the 3rd total number (the large number), here the “3”, 200 more Generated invoices (so that in the 3rd column 200 small products, analogous to the large totals line) begin with “317”, suggests the assumption that very large numbers will also come into play with this algorithm: 20 “externally” not apart Alternatives to be kept with 2-digits, 200 with 3-digits, 2,000 with 4-digits, 20,000 with 5-digits, etc. Already at the 10th position (digit, column) it would then involve 2 billion calculations. This state, extrapolated to 500 or even 1,000 or 2,000 digits, requires the previously common “brute force” methods that nano computers long for and, until they are used, conventional large-scale computing systems (which, for example, the NSA seems to have, which is why she was able to report: “Up to 500-digit numbers are no problem for us”) with her restless calculation, which then collapse according to what is said at about 2,000 digits.

With the additional use of “language logic” (so called to emphasize the importance of independent thinking, also including language), a solution for an easier procedure may be offered here:

If the target number (ie the present large number; here in the example it is 51,353,317) namely after processing each column (each with a new pair of factor digits for the multi-digit prime factors sought) through the results of the individual to Then after a few steps (these are the relatively smaller, the larger the sought prime factors), increasingly more and more, crystallize out the only “correct” example for the present concrete example.

The calculation 43 \* 19 = 817, which is known here as correct, for example, does not give a clear result when divided by the target number (i.e. the total line, i.e. the large number: 51,353,317), but it does so with 3 digits (643 \* 719 = 462,317 ) here results in the division 51.353.317: 462.317 = 111, ..., so a strong orientation towards one (1), which is obvious. Because results that adapt to a target value result in a 1.0 at the latest if they are equal, if they are divided by themselves.

This approximation to “1” is not a percentage calculation, but represents a relation, a ratio to each other, a factor. This factor is “1” if one number is identical to the other number, and it approaches this factor “1” the closer the sought and building up (eg counted) prime factor is to the actual multi-digit prime factor within the large one Number is coming.

For a more precise calculation of the mutual proximity of these two aforementioned numbers, it is advisable to bring these two numbers “at eye level”, i.e. to make them approximately the same size. For this purpose, digits should be cut off on the right side of the large number.

Compared to all the alternatives given here, none of the other calculations with results of 317 comes closer to “one” than the calculation 643 \* 719, which here turns out to be more and more correct = 462,**317**.

Without having to get to the bottom of this phenomenon, it is sufficient to state that all results at this point show “317”, but have various “attachments”. These appendices consist of the same parts (described in the following point), which only have different numerical values ​​and accordingly sometimes carry and sometimes not, but do not have to be "understood internally". It is sufficient to assume that the compound “317” and the appendix allow a meaningful statement that arithmetic mathematics is able to recognize, with which it can present the “winner” relatively early and prevent large amounts of calculations.

**What connects the two pairs of digits from the list to be completed next with the new third column?**

The 3rd column is made up of two number ranges: the so-called overhangs and the two numbers (head and 1st line body digit).

The so-called overhangs are the digits and numbers that would normally move into the unit digits of the next column on the left (here the 3rd column) if there are numbers on the corresponding lines. Specifically, these would be so-called transfers

a) part of the units digit of the so-called "breast" (i.e. the 3rd digit of the counted prime factor in the 1st line),

b) the units digit of the product in the 2nd line from the known 2nd numerator (here “1”, see the second counting factor number “19”) multiplied by the known, because constant, “head” (here “4 ”), Only in the 3rd column because the 2nd line is shifted one place to the left and now, since a 3rd line is added in the 3rd column, it becomes the removable middle section

and c) if applicable and usually the total amount (here “1”).

Insert: When removing any middle section, you must ensure that any decimal places that may be present in each middle section number can only be subtracted in the next column to the left.

This so-called overhang is shown in its sum (1- or 2-digit) in the product of the two 2-digit factors in its so-called appendix with a single sum and can therefore, without having to add up the summands individually, can be deducted from the 3rd total line! In the correct example here (43 \* 19 = 817) the overhang after the 2nd column is “**8**” (cf. **8**17) and is made up of “3”, “4” and “1”. Overall, this is the so-called overhang, which can be deducted from the 3rd total number “3”. The alternative invoices also have 2-digit excesses that are deducted across two columns. An 83 \* 99 = **82**17 requires subtracting the “**2**” from the present 3rd total number and the “**8**” from the next 4th total number.

After deducting the above-mentioned excess at the correct total number (here both the 3rd and sometimes the 4th total number), a digit or number remains with which a separate double row is formed. Since the overhangs of all “17” and “317” numbers are different, almost all subtractions also look different, so they are calculated with different column numbers (in the 3rd column below each other).

In the calculation that is correct here as an example, the third total line “3” minus the above-mentioned overhang “8” results in a “5” and requires a double row of 15. If this “5” is not calculated in the aforementioned way, but is considered individually in the 3rd column, then it is made up of “4” and “1”. The “4” is the one digit of the “54”, the result of the multiplication of the known first numerator (the “9”; see prime factor “1**9**”), which is constant for the first line, by the previously unknown 3rd digit of the counted number Factor (the so-called “breast” here, here “6”) arose and only became “57” with the initially mentioned transfer “3” (and below in the total line to “3”) and the “1” is the one Digit from “21 = 3 \* 7), where the“ 3 ”on the left is the known and constant“ head of the counted factor and the “7” on the right is the new, third counting factor that was not yet known (see the 3 Prime numerals “**7**19”).

These two summands of the 3rd column: 4 and 1 (or 54 and 21) are characterized by the fact that they are the only two numbers here in the 3rd column (and in each further) (there are per column, even if this 1000 summands contains only exactly **two lines**, for which this applies: The first line and the newest, lowest!) which have only one known factor as a product (above the constant first digit (here “9”) and below the constant first number (the so-called “head”; here 3).

In order to find these numbers, in spite of their missing factor, all 10 possible possibilities in each column must be produced and checked by multiplication.

The above transfers are also subtracted out because they are not required. Only numbers without transfers are used for factoring.

The transfers (decimal places that move to the left) arise from multiplication, not from factoring. They are only important when factoring in order to know how they have changed the individual summands of each column or the corresponding sum figures.

If the transmissions are completely removed everywhere, they can stay away. They are no longer needed to the left.

**Concrete examples:**

The concrete example here was the multi-digit prime factors 7643 and 6719. Your product is 51,353,317. In the practice of factorization, however, these above-mentioned only correct prime factors are not known.

When using the list presented in the next article and the numbers on which it is based, there are 20 possibilities (in the case of “17” as the first total number) that show this “17”. These 17 starting digits can come about in 20 different ways (33 \* 49 or 43 \* 19 or 63 \* 59 etc.).

For reasons of space and because the procedure is always the same, i.e. it can be processed very easily by an algorithm, the illustration here is reduced to a few examples.

Of the first 20 cases from the list from Part 10, only 9 are mentioned here and only 2 examples of the 10 “sub-calculations” per case are mentioned.

(1)

93 \* 69 = 54**17**

(2)

83 \* 99 = 82**17**

(3)

73 \* 29 =21**17**

(4)

63 \* 59 = 37**17**

(5)

53 \* 89 = 47**17**

(6)

43 \* 19 = 8**17**

(7)

33 \* 49 = 16**17**

(8)

23 \* 79 = 18**17**

(9)

13 \* 09 = 1**17**

To (1):

Here the “54”, distributed over 2 columns (in the 3rd column comes the “4” and in the 4th column the “5”), represents the so-called appendix 3. Subtracts the total line “3” (and then the “5” from the 4th total number “3”). The first subtraction 3 - 4 is 9.

From this 9 the double row 0 + 19 is formed:

0 1 2 3 4 5 6 7 8 9

19 8 7 6 5 4 3 2 1 0

To (2) to (10):

Each of the above calculations requires its own double row depending on the digits of the overhang.

The appendix “82” under point (2) causes a double row of 11 for the 3rd column according to 3 minus 2 and a row of 5 for the 4th column 3 - 8.

The appendix “21” under point (3) causes a double row of 12 for the 3rd column according to 3 minus 1 and a row of 11 for the 4th column 3 - 2.

The appendix “37” under point (3) causes a double row of 16 for the 3rd column according to 3 minus 7 and a row of 10 for the 4th column 3 - 3.

The appendix “47” under point (3) causes a 16-row double row for the 3rd column according to 3 minus 7 and an 11-row row for the 4th column 3 - 4.

The appendix “8” under point (3) causes a 15 double row for the 3rd column according to 3 minus 8. The overhang does not have a second position here.

The appendix “16” under point (3) causes a double row of 17 for the 3rd column according to 3 minus 6 and a row of 12 for the 4th column 3-1.

The appendix “18” under point (3) causes a double row of 15 for the 3rd column according to 3 minus 8 and a row of 12 for the 4th column 3-1.

The appendix “1” under point (3) causes a 12 double row for the 3rd column according to 3 minus 1. The overhang does not have a second position here.

**The 3rd factor and sum digits**

If the above and other 2-digit prime factors are to be given a third factor pair (which gives them all a third total number of “3”, so that all 200 calculations will have the results “317” as the initial digits), then the so-called here goes Castell-fact algorithm as follows:

The above example 93 \* 69 (= 5417) resulted in 3 - 4 = 19. And this 19 double row shown above had 0 + (1) 9 as the first superimposed double digit. Algorithmically this means:

x 9 3 \* y 6 9 result in 0 9 3 \* 3 6 9, because the 0 \* 9 = 0 is at the top and the 3 \* 3 = 9 at the bottom.

Correspondingly, 93 \* 69 becomes 993 \* 669 (= 664.317) with the help of the “1” above and the “4” below from the 19 double row,

with “2” at the top and “3” at the bottom you get 893 \* 969 (= 865.317),

with “3” at the top and “2” at the bottom results in 793 \* 269 (= 213.317),

with “4” at the top and “1” at the bottom, you get 693 \* 569 (= 394.317),

with “5” above and “4” below results in 593 \* 869 (= 515.317),

with “6” above and “9” below results in 493 \* 169 (= 83.317),

with “7” at the top and “8” at the bottom, you get 393 \* 469 (= 184.317),

with “8” at the top and “7” at the bottom results in 293 \* 769 (= 225.317),

with “9” at the top and “6” at the bottom, the result is 193 \* 069 (= 13,317).

Corresponding to the above 93 \* 69, you can proceed in the same way with 83 \* 99 (here the double row 13 - 2 = 11), or with 73 \* 29 (double row 13 - 1 = 12), or with 63 \* 59 (double row 13 - 7 = 16), or with 53 \* 89 (double row 13 - 7 = 16), or with the (correct here) 43 \* 19 (double row 13 - 8 = 15), or with 33 \* 49 (double row 13 - 6 = 17 ), or with 23 \* 79 (double row 13 - 8 = 15), or with 13 \* 09 (double row 13 - 1 = 12) etc.

**Concrete numerical examples for the above mentioned designs**

Here arbitrarily selected 7 of the total of 20 invoices, which come from the list in the next part (eighth part of the article) for the case where the results start with “17”:

93 \* 69 83 \* 99 73 \* 29 63 \* 59 53 \* 89 43 \* 19 33 \* 49

837 747 657 567 477 387 297

558 747 146 315 424 43 132

5417 8217 2117 3717 4717 817 1617

In each of the 2nd columns of the aforementioned calculations are 3 + 8 one above the other and 4 + 7 and 5 + 6 and 6 and 5 and 7 + 4 and 8 + 3 and 9 + 2 and result in the 2nd total number “1” (or 11). For the other bills, however, the “9” is used as the 2nd total number. And the pairs of digits in the 2nd column, one above the other, supplied by the double row have already deducted the transfer “2” (from the 1st column). They are namely 1 + 8, 2 + 7, 3 + 6, 4 + 5, 5 + 4, 6 + 3, 7 + 2.

Factoring can only be carried out with these already “cleared” double totals, which are available for addition as single digits on top of each other, but are all in all numbers that mostly have decimal places.

The first-mentioned 1, which is actually an 81 in a 9-ary row (see the 9 on the right) as the second counted factor generates the “9” on the left in the first example as a new factor. Accordingly, the “8” below stands for an 18 in a row of 3 (see the “head” 3 for the counted factor 93) and creates 6 as the new counting factor.

The same applies to the other examples.

As described above, the third total digit (or 3rd digit of the large number) consists of a “3”, but the double rows used by the above 7 examples calculate according to the numerical value of the so-called overhang (i.e. the 1 to 2 digits outside the “17”) with different double rows. In the first example, the numerical value outside the “17” consists of “54”, so that the “4” for the 3rd total digit and the “5” for the 4th total digit are deducted from the respective total digits. The required double row for the 3rd column is (13 - 4) a 19 double row.

In the second example, the numerical value outside the “17” consists of “82”, so that the “2” for the 3rd total digit and the “8” for the 4th total digit are deducted from the respective total digits. The required double row for the 3rd column is (13 - 2) an 11 double row.

In the third example, the numerical value outside the “17” consists of “21”, so that the “1” for the 3rd total digit and the “2” for the 4th total digit are deducted from the respective total digits. The required double row for the 3rd column is (13 - 1) a 12 double row.

In the fourth example, the numerical value outside the “17” consists of “37”, so that the “7” for the 3rd total number and the “3” for the 4th total number are deducted from the respective total number. The required double row for the 3rd column is (13 - 7) a 16 double row.

In the fifth example, the numerical value outside the “17” consists of “47”, so that the “7” for the 3rd total digit and the “4” for the 4th total digit are deducted from the respective total digits. The required double row for the 3rd column is (13 - 7) a 16 double row.

In the sixth example, the numerical value outside the “17” consists of “8”, so that the “8” is deducted for the 3rd total number. The required double row for the 3rd column is (13 - 8) a 15 double row.

In the seventh example, the numerical value outside the “16” consists of “37”, so that the “6” for the 3rd total digit and the “1” for the 4th total digit are deducted from the respective total digits. The required double row for the 3rd column is (13 - 6) a 17 double row.

**Finding the only correct calculation**

At the top there are now 7 of 20 examples, all of which add "17" to the sum and which must all be treated equally, i.e. calculated and factored, since it is not known which of these 20 calculations is correct for the underlying example.

The already proposed method of establishing the relationships between the partially reconstructed prime factors and the target size, i.e. approximately half of the large number, does not yet work here. The calculation 43 \* 19 = 817, which is already known here as correct, is still too conclusive with its overhang “8”.

In this example (43 and 19), too, of which, as an exception, it is known beforehand that it is the right one with regard to the specific example (total line 51.353.317), 10 alternatives have to be created in order to produce its two 3rd factor numbers.

43 \* 19

. . . . . . . . .

(3) 6 7

4 3

. . . . . . . . .

8 1 7

The overhang “8” causes the total number to be corrected from “3” to “5”. For this total number “5”, the 10 alternatives are then produced using the double row:

0 1 2 3 4 5 6 7 8 9

15 4 3 2 1 0 9 8 7 6

In the following, the new third digits are inserted into the previous 2-digit invoice “43 \* 19”.

This is done using the digits 0 + 5, 1 + 4, 2 + 3, 3 + 2, 4 + 1, 5 + 0, 6 + 9, 7 + 8, 8 + 7 and 9 + 6 listed one above the other .

The “insertion” is done in such a way that the relevant digit above (here first the “0”) is the units place of the product that results from multiplying the constant first digit “9” with (in the 9th row here) the still unknown third number (the “breast”) results. Since “9” multiplied by “0” gives the result “0”, the 3rd counted factor found is a “0”.

Correspondingly, the bottom 5 (in the 3-way row here) is the unit's digit of the product that results from 3 times the still unknown third digit, i.e. 15, which results in the factored digit 5. The product of **0**43 \* **5**19 = 22,317.

Correspondingly, in the second 43 \* 19 example, the second double-row digits 1 + 4 are inserted, in that the “1” represents the units digit of a product that includes the first digit “9” in a 9-digit row with an as yet unknown numbered third Digit multiplied. The question could be: Which product with a “1” as the one digit needs which new factor for its multiplication, if one of the two factors is “9”? Or even simpler: Which number with a “1” as the one digit indicates the 9-digit series, and which new factor results when this number is divided by “9”?

The 9-er row: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90 The new counted factor is “9” and arose from 81.

And below, too, the new number factor “8” was created from 24 by means of the lower double row number “4”.

The 24 can be found in the 3-way row: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30. Only the “24” has the “4” as a unit number and results from the known number and constant “head” factor “3” divides the above “8.

The product of **9**43 \* **8**19 = 772,317.

Accordingly, the now 3-digit factors arise **8**43 \* **1**19 = 100,317; **7**43 \* **4**19 = 311,317; **6**43 \* **7**19 = 462,317; **5**43 \* **0**19 = 10,317; **4**43 \* **3**19 = 141,317; **3**43 \* **6**19 = 212,317; **2**43 \* **9**19 = 223,317; **1**43 \* **2**19 = 31,317.

**Result of the previous calculations**

There are 20 examples from the list created in the 10th part, which although different “overhangs” (i.e. 1 to 2 additional digits), but all equally have the first additional digits “17”.

Since it is already known in advance in the article which of these 20 invoices is the correct invoice belonging to the given specific example (namely the 43 \* 19 = 817 or 7643 \* 6719 = 51.353.317), the anticipated note that only the aforementioned correct calculation fulfills the condition required in the 11th article of the closest “relation” to the target number. The differences are not significant at the beginning (with 2-digit factors), but then become more and more so with each further step. In the present example it was already clear after the 3rd column (because of the shortness of the numbers used) who was closest to some of these “**1.0**”.

The aforementioned results (22.317, 772.317, 100.317, 311.317, **462.317**, 10.317, 141.317, 212.317, 223.317 and 31.317) clearly show (without the need for a mathematical test) that only the number 462.317 of the target and target number 51.353 .317 comes close. The declination is still approx. 10% if 51.353.317: 462317 = 111.0 ... or (if the number of digits of both numbers, capping the digits from the right, is adjusted) 513533: 462317 = 1, 110 ... ..

Comparisons with the other 20 “17” invoices from the list confirm that (from the 3rd column onwards) none of the other invoices match the values ​​of the correct invoice.

Further tests with other numerical examples also give the same result: The calculation with the closest relation to the target number is the correct one for the example.

**Alternative approach**

The idea of ​​finding the correct calculation based on a factor close to “1” from the 20 to 30 offers initially supplied by the database with the correct first two total digits (in the examples given here “07” or “17”) came about here, for example with other numbers, not further checked for their correctness, since with their many necessary attempts they would in any case be a time-consuming and unsafe procedure.

The following procedure is more obvious and clearer: The first two total digits are processed column by column or using the “1,000 bills” in the 10th part, which shows the first two columns as a whole.

In both cases, the 3rd and possibly 4th digits in the 3rd column, which were created by multiplying the first two 2-digit factor digits to the left, are subtracted from the real 3rd digit of the large number (this was the case with the 17-example example from 817 the 517, since the present sum digit is “3”.

For the sake of convenience, the “817” resulting from the multiplication represents the total transfer from the 2nd column to the 3rd column with its “8”. This total transfer from the 2nd to the 3rd column is made up of 3 parts: a ) The transfer in the 1st line (here at 43 \* 19 the 38, which transfers its decimal place “3” to the 3rd column. Since this is about the results of two 2-digit numbers, it is in the 1st line There is no one digit in the 3rd column that could hide this transfer “3”. The transfer “3” is therefore only in the 3rd column and is therefore visible. b) The transfer in the total line (here “1”), from the 2nd to the 3rd total number, as the 2nd column contains the column numbers 38 + 3 or the units numbers “8” and “3” and therefore comes in the 10-digit range, which has its decimal place “1 ”In the 3rd column. c) And in between the number “4”, which still belongs to the calculation of 2, but is already in the 3rd column because of the successive shifting to the left of the following lines. The sum of these 3 digits mentioned above results in the total transfer from the 2nd to the 3rd column (here “8”) and, after subtracting them from the actual 3rd total number “3”, the “5”.

This difference (here the “5”) is able to show the two numbers that actually determine the 3rd column. Here are these two numbers “54” from the 1st line and “21” from the last, bottom line. They result from the sum 75, which is included in the list to be drawn up at the end of this article and which names the two aforementioned figures.

In fact, the 75 “visible” is not present, but only the new 3rd total number “5”. In order to know what the decimal place of the aforementioned “5” is, the two transfers from the 3rd column must be recorded and added (these are logically the already described digits “3” (decimal place of 38) and the “4” (the because of the shift to the left of the 2nd line in the 3rd column).

In order to have a clear view of the transfers in the new column on the left, new multiplications have to be carried out after each column (including their factoring), in which the two factor numbers have each time been expanded by one digit. The multiplication of the 2-digit factors leads to the desired results in the 3rd column. The multiplication of the 3-digit factors explains the relationships in the 4th column (here the number in the 2nd row becomes the so-called middle part, which also has to be subtracted. In the 5th column, these are the 2nd and 3rd Line, in the 6th column the 2nd, 3rd and 4th lines etc.). And the multiplication of the 4-digit factors gives information in the 5th column, etc.

Concrete:

Specifically, the multiplication of the two 2-digit factors results in 43 \* 19 = 817.

The multiplication of the two 3-digit (each additional digit is found after factoring the previous column) factors 643 \* 719 = 462317.

The multiplication of the now 4-digit factors 7643 \* 6719 = 51353317 etc.

**A helpful list**

Since the digits of the multi-digit counting factors usually alternate, each line within the calculation has its own counting factor.

As can be seen in the above specific example, the first digit represents a “9”. The first line (it is the most important of all lines because it is the only place where the required factor digits of the counted, multi-digit factor arise) is a 9 -er-row, ie it only consists of numbers that result from “9” (the fact that only their units are visible does not change this fact. The top, first line always consists of the 10 possibilities 0, per column, 9, 18, 36, 45, 54, 63, 72, 81).

Although the counters of the lines below are not known and usually change (in this example it is 9, 1, 7 and 6, but written from right to left) the vertical line (diagonally from right to left) can contain 3 -Row, because it only appears again with its “head”, ie with its first digit in the bottom line, and this head is a “3” in this example. The row of 3 is 0, 3, 6, 9, 12, 15, 18, 21, 24, 27.

For this case (as numerator the “9” and as counted digits the “3”) the invisible decimal places would always be easy to deduce. A “5” in a row of 3 can only be a “15”, a “9” in a row of 7 can only be a “49” and a “6” in a row of 9 only a “36” (which can be factored into 4 \* 9) etc.

In the example chosen here, the mentioned “list” would be a matrix that represents a row of 9 at the top in the horizontal line and a row of 3 at the beginning of the vertical line on the left. In between in the columns are the sums of the two reference numbers on the edge. With a look at one of the sums in the matrix, the associated summands would be readable at the edge.

**0 9 18 27 36 45 54 63 72 81 (90)**

**3** 12 21 30 39 48 57 66 75 84 93

**6** 15 24 33 42 51 66 69 78 87 96

**9** 18 27 36 45 54 63 72 81 90 99

**12** 21 30 39 48 57 66 75 84 93 102

**15** 24 33 42 51 60 69 78 87 96 105

**18** 27 36 45 54 63 72 81 90 99 108

**21** 30 39 48 57 66 75 84 93 102 111

**24**  33 42 51 60 69 78 87 96 105 114

**27** 36 45 54 63 72 81 90 99 108 117

**(30)** 39 48 57 66 75 84 93 102 111 120

This approach has not yet been tested in practice either. But it could be the most effective approach so far, because you do not have to calculate in every further column with 2 to 3 further double rows with 10 possibilities each in the associated uncertainty of having made the right choice.

The search number within the aforementioned list reliably determines the two numbers searched for each time (because there are only exactly 2 numbers in each column after calculating the middle parts that have to be searched for and factored), and the required decimal place of the matrix search number results from the mentioned unfinished last column on the left.

More detailed investigations, especially with the order (because it is pointless to use decimal places to determine numbers in the right column beforehand, which already exist and were only able to provide these decimal places), are still being carried out.

**Further questions**

What makes this list, to be presented in more detail in the eighth part of this essay, unclear is that there are on average 3 possibilities for each sum figure requested within the matrix (with “5” e.g. 15, 45 and 75) and often different offers within one possibility of summands. For example, the list provides four possible additions for the total number “5” for the total number “75”: 45 + 30, (the correct one in this example) 54 + 21, 63 + 12 and 72 + 3.

So the question to be solved here is to find out how the correct alternative can be found in each case.

This question is by far the most important of all the questions asked so far. Because every single digit of the large number is only about 2 numbers (products), of which only 1 factor and a common sum is known.

The list presented in the next part concerns the fact that the top number of the respective column is an adjusted number from the 9-row and the row of the “heads” is practically a 3-row.

It is clear that other combinations of prime end numbers are also possible: 1, 3, 7 and 9 can be combined with “1” or all of these can also be combined with themselves and with each other, as with 3 with 7, 3 with 9 , 9 with 7 and all this again in reverse order.

But since precisely this one small question (here in the example: “Which two column numbers form the total number“ 5 ”?) Is the last and all-important one, it makes sense to move away from all previous attempts that have not yet been able to answer this question and to check whether a definitive solution can be found here with this type of list.

Since complete numbers are necessary for factoring (here in the example these would be “54” (6 \* 9) above in the 1st line and “21” (7 \* 3) below in the last, here third, line), and Since these two numbers are supplied completely and in 4 variations from the present list, this list should be examined more closely and it should be clarified whether the two only correct summands can be determined for a respective calculation example.

The said list adds up inside the numbers assigned to the selected sum in the two rows of 9 and 3 that are visible on the outside. Both rows begin with the common number 12 (9 + 3). This 12 then continues horizontally and parallel to the row of 9 to the right in steps of 9 and vertically and parallel to the row of 3 downwards in steps of 3, and ends after 10 lines of 10 numbers at 120 (90 + 30).

With the 4 pairs of summands 45 + 30, 54 + 21, 63 + 12 and 72 and 3, it is noticeable that the first numbers come unchanged from the 9-series and the second summands from the 3-series. Whenever the number on the upper left side increases by 9, it decreases accordingly on the lower right side. The latter is conspicuous, since 9-step steps are normal in the 9-step row, but not in the 3-step counter-clockwise counting down row, which also counts down in 9-step steps.

In both cases, one can deduce the corresponding adjusted decimal digits from the units digits (“4” can only mean “45” in a 9-digit row, “1” in a 3-digit row only “21”, etc.). And because of the fixed common total number (here “5”) one can deduce the other number from a summand number. But it is precisely these two column numbers (here “4” and “1”) that cannot be identified and recorded with the methods developed here so far.

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